



Stochastic Sensitivity Testing for the Canada Pension Plan Actuarial Reports

Assia Billig,
Office of the Superintendent of Financial
Institutions, Canada



INTERNATIONAL SOCIAL SECURITY ASSOCIATION
ASSOCIATION INTERNATIONALE DE LA SÉCURITÉ SOCIALE
ASOCIACIÓN INTERNACIONAL DE LA SEGURIDAD SOCIAL
INTERNATIONALE VEREINIGUNG FÜR SOZIALE SICHERHEIT



TECHNICAL SEMINAR FOR SOCIAL SECURITY ACTUARIES AND STATISTICIANS
“Sustainable financing and extension of social security coverage”



Montevideo, Uruguay, April 27-28, 2010

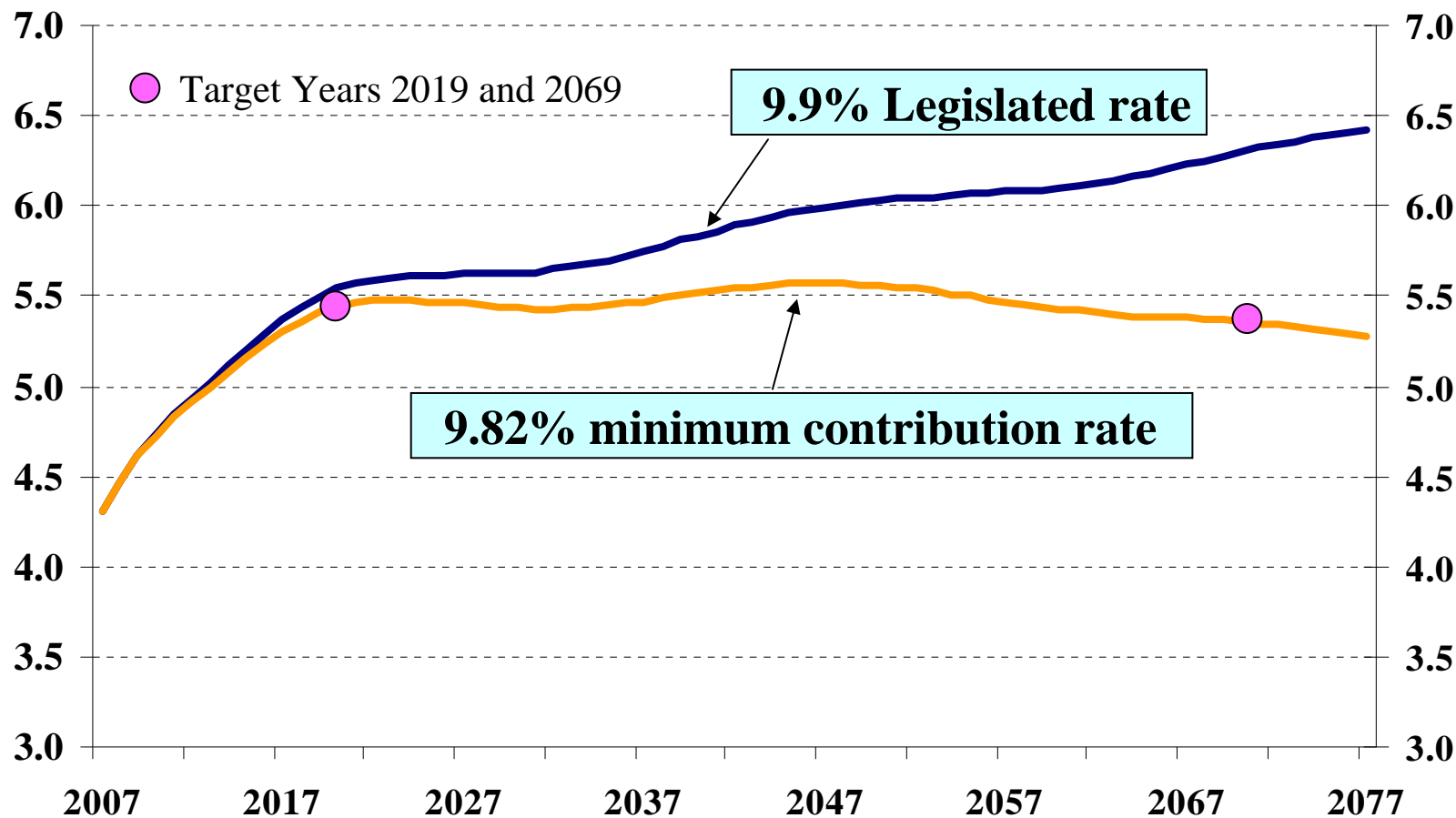
CPP, a Partially Funded Defined Benefit Pension Plan

Canada Pension Plan – a part of the Second Tier of the Canadian System

- Defined Benefit Plan (replacement rate of 25% up to a specified maximum)
- Partial funding through employee/employer contributions.
 - Steady-state financing goal: stabilize and minimize the contribution rate.
 - Steady-state contribution rate: lowest rate that can be charged that is sufficient to sustain the Plan without further increase.
 - Main indicator – stable assets/expenditure ratio
- Full funding of future benefit improvements.

Under best-estimate assumptions and minimum contribution rate of 9.82%, A/E ratio stabilises at around 5.5

Evolution of the Asset/Expenditure Ratio



Uncertainty of results – an important part of the reporting

- Statutory actuarial reports of the CPP are produced every 3 years
- The projections are done for the period of 75 years
- Significant number of assumptions is required

It is important to address the uncertainty of results

- CPP#23 report contains a combination of:
 - Scenario testing: younger/older population, economic half-cycle, financial market volatility
 - Individual assumptions sensitivity tests: deterministic or stochastic

Scenario and deterministic tests

Scenario tests

- Due to the complex dynamics of assumptions correlation, stochastic outputs of individual tests could not be easily combined
 - Younger and older population scenarios illustrate potential sets of demographic and economic assumptions
- Short-term impact of economic cycles
 - Economic half-cycle scenario and financial volatility scenarios
- Elements of stochastic analyses are involved in scenarios' development

Deterministic tests

- Retirement rates for cohort at age 60
 - Insufficient experience – early retirement provision was introduced only in 1987
- Participation rates and unemployment rate
 - Past workforce data do not reflect future trends such as expected labour shortage due to aging of the population

Stochastic sensitivity tests are providing report's users with likelihood of potential outcomes

- It is unrealistic at this time to make the whole CPP valuation model fully stochastic
- Stochastic modeling is incorporated in determination of alternative assumptions for sensitivity analysis
 - Project probability distribution of potential outcomes
 - High cost and low cost assumptions
- **Word of caution:** stochastic modeling results are dependably on
 - model selection
 - period of past experience chosen
 - accuracy of assumptions regarding probability distributions and correlation between variables
 - length of projection period

Stochastic models should not be used to confuse readers!

Office of the Chief Actuary (OCA) stochastic tests are based on ARIMA(p,d,q) models

- Autoregressive Integrated Moving Average - general class of time series forecasting models
 - d = order of differencing
 - p = number of Autoregressive (AR) terms
 - q = number of Moving Average (MA) terms
- For example: ARIMA(1,1,1)

$$\hat{Y}(t) = \mu + Y(t-1) + \varphi \times (Y(t-1) - Y(t-2)) - \theta \times e(t-1)$$

where:

- $Y(t)$ is a random process,
 - μ is a mean term
 - $e(t)$ is a random error
- Variations:
 - It is also possible to fit transformed data (e.g. Log ARIMA)
 - If $d=0$ and $q=0$, the model is referred to as $AR(p)$
 - If only $d=0$, the model is referred to as $ARMA(p,q)$

OCA model – combination of stochastic and deterministic elements

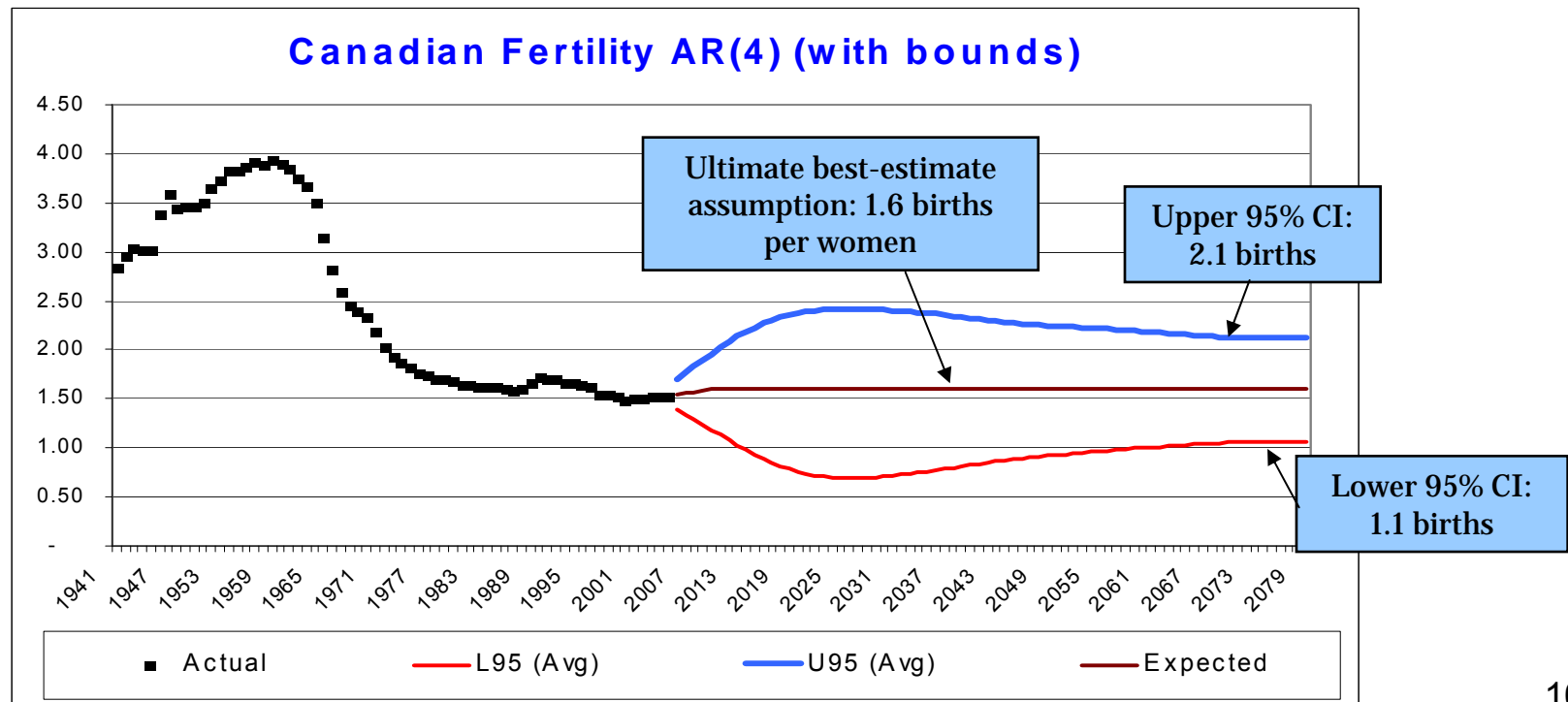
- **SAS** (*Statistical Analysis System*) - fit models to historical data
 - Model is chosen based on the goodness-of-fit statistics and model's simplicity
 - Estimated parameters are used for projections
- **Excel (VB macro)** – run 1,000-20,000 stochastic scenarios per run depending on number of variables
 - Each scenario generates random error using normal distribution with zero mean and variance based on past experience
 - Deterministic element - center predictions around best estimate
 - Stochastic scenarios generate confidence intervals and cumulative average confidence intervals centered around best estimate

Model variations for correlated assumptions

- If assumptions are correlated it is possible to:
 - Fit each assumption separately, but assume errors are correlated
 - **Cholesky decomposition** is used to correlate error terms
 - Example of use: mortality rates by age group
 - Use Vector Autoregressive Modeling (VAR)
 - Joint modeling of time-series processes
 - Data indicates that these variables may be related to each other's past values
 - In stochastic forecast, each estimated variable depend on previous values of all variables
 - Errors are correlated using Cholesky decomposition
 - Example of use: Rate of return on assets classes and inflation

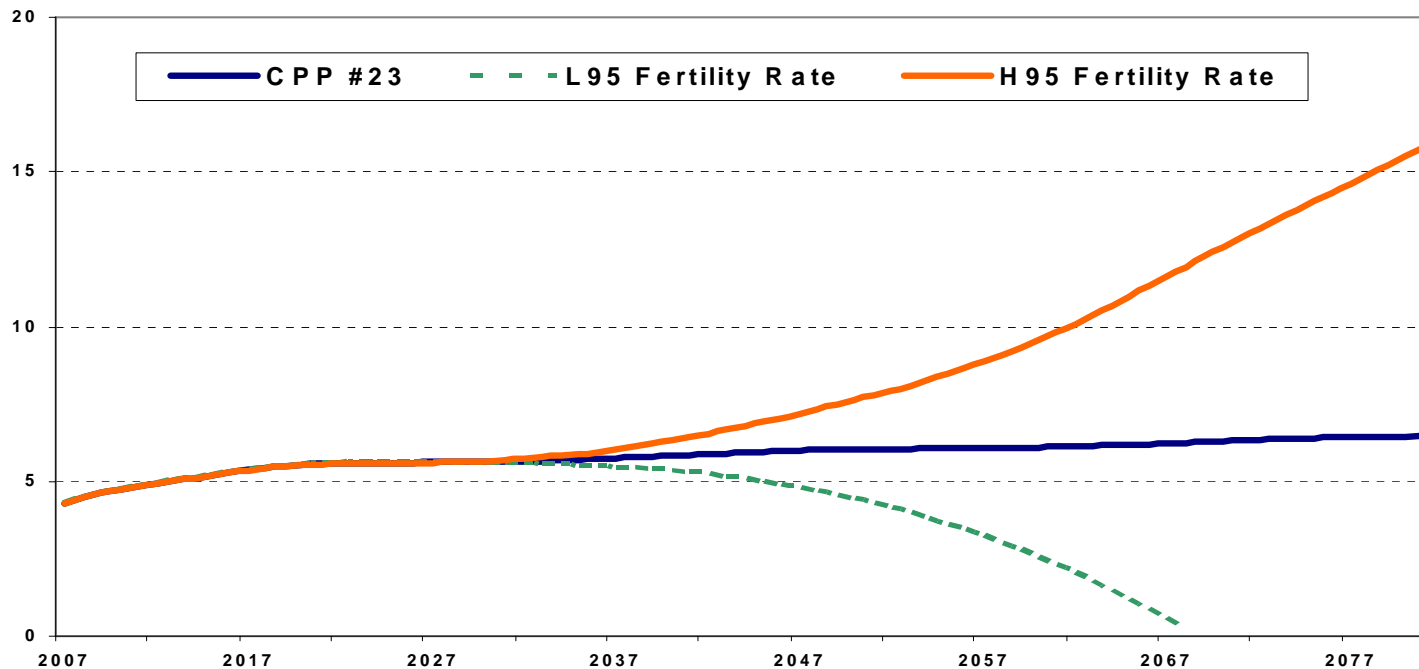
Fertility – 95% confidence interval

- Years 1941-2079 (years 2006-2079 are projected)
- AR(4) model was chosen: simple, good fit, does not reach upper and lower bounds



Under lower fertility scenario assets are depleted by 2069

Evolution of the Asset/Expenditure Ratio (9.9% Contribution Rate)



The minimum contribution rate falls between 9.25% and 10.45%

Mortality model output: confidence intervals for life expectancies

- 40 Age Groups correlated using Cholesky decomposition:
 - Male and Female / Under 1, 1-4, 5-9, 10-14,...85-89, 90+
- Initially tried to fit Mortality Improvement Rates
 - Poor fit statistics → look at Mortality Rates (deaths per 1,000)
 - Model Mortality Rates and convert back to improvement rates
- Log ARIMA(0,1,0) model produced good fit for all age groups (except 90+)
- Years 1926-2079 (years 2005-2079 are projected)
- 95% confidence intervals and medians for mortality rates, mortality improvement rates and life expectancies at birth and age 65

Sensitivity test using stochastic process

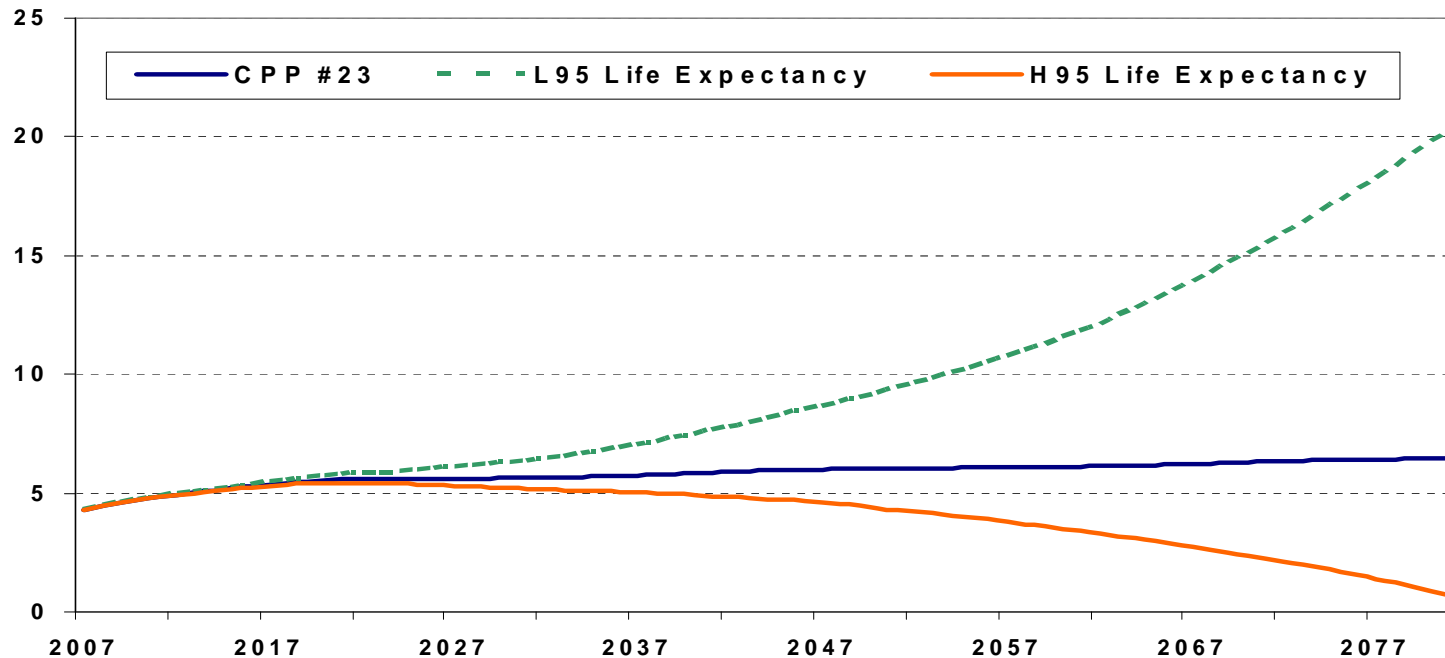
Canadian Life Expectancies, with improvements after the year shown

At birth		CPP #23	Lower 95% CI	Upper 95% CI	Range
Males	2050	87.4	80.8	91.3	10.5
Females	2050	90.2	81.9	94.6	12.7
At 65					
	2007	19.3	17.6	20.7	3.1
Males	2025	20.6	17.7	22.9	5.2
	2050	21.9	17.8	25.1	7.3
	2007	22.0	19.5	24.0	4.5
Females	2025	23.0	18.9	25.9	7.0
	2050	24.2	18.6	27.9	9.3

†3

Although mortality rates could not decrease forever, their future trajectory could require an increase in the contribution rate

Evolution of the Asset/Expenditure Ratio (9.9% Contribution Rate)



The minimum contribution rate falls between 9.2% and 10.2%

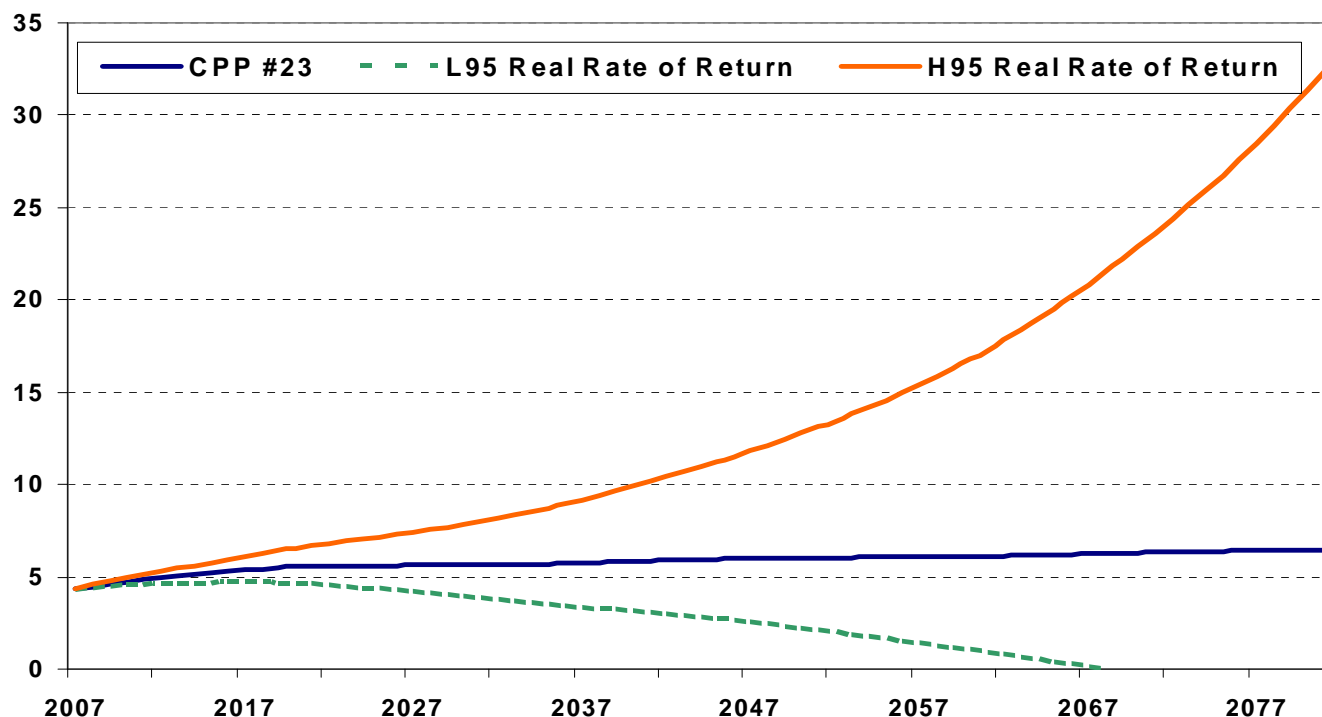
Real rate of return of portfolio – four correlated variables

- Rates of return of Canadian equities, foreign equities and fixed income as well as inflation are simulated using 4VAR(2) model
 - Each variable at time t depend on values of all four variables at times $(t-1)$ and $(t-2)$
 - Years 1938-2079 (years 2007-2079 are projected)
 - Errors are correlated using Cholesky decomposition
- Projections are centered around best-estimate
- Nominal return on portfolio is calculated using CPP#23 portfolio assets allocation and then converted to real rate of return of portfolio

Lower 95% CI	Best Estimate	Upper 95% CI
2.7%	4.2%	5.7%

Real rate of return sensitivity testing exhibits the highest volatility in the contribution rate...even for a partially funded plan

Evolution of the Asset/Expenditure Ratio (9.9% Contribution Rate)



The minimum contribution rate falls between 9.0% and 10.7%

Conclusions and next steps

- The application of stochastic processes is a long journey, not a destination
- Higher processing power of computers allows development of more sophisticated models
- CPP#23 peer reviewers recommendation:
 - “We recommend that the Chief Actuary maintain the tradition of continual improvements to actuarial methods by such actions as
 - applying more forward-looking stochastic analysis, and
 - developing more plausible and consistent sensitivity tests for key assumptions.”
- Explore possibility of simultaneous projections of fertility and migration rates



Stochastic Sensitivity Testing for the Canada Pension Plan Actuarial Report

Appendix



INTERNATIONAL SOCIAL SECURITY ASSOCIATION
ASSOCIATION INTERNATIONALE DE LA SÉCURITÉ SOCIALE
ASOCIACIÓN INTERNACIONAL DE LA SEGURIDAD SOCIAL
INTERNATIONALE VEREINIGUNG FÜR SOZIALE SICHERHEIT



TECHNICAL SEMINAR FOR SOCIAL SECURITY ACTUARIES AND STATISTICIANS
“Sustainable financing and extension of social security coverage”

Montevideo, Uruguay, April 27-28, 2010

Cholesky Decomposition

- A Cholesky Decomposition is the process of decomposing a positive definite symmetric matrix \mathbf{V} into the product of two matrices \mathbf{L} and \mathbf{L}^T ($\mathbf{V} = \mathbf{L}\mathbf{L}^T$). \mathbf{L} is a lower triangle matrix, which makes its transpose \mathbf{L}^T an upper triangle matrix.
- Cholesky decomposition is used to convert random standard normal error terms to a multivariate normal distribution with mean = 0 and a variance-covariance matrix \mathbf{V} obtained using historical data.
- By multiplying the Cholesky matrix \mathbf{L} by the vector of error terms, it is possible to obtain a vector with the required multivariate normal distribution. The matrix \mathbf{V} is the variance-covariance matrix of residuals, the difference between the actual historical data points and the estimated data points (using the chosen ARIMA model).

Log ARIMA (0,1,0) model for mortality rates

- **1st difference:** assumes mean is time-varying
- **Log transformation:** allows mortality rates to approach, but not hit, zero

The general form of the equation used is :

$$\ln(Y_{k,t}) = \ln(Y_{k,t-1}) + \mu_k + \varepsilon_{k,t}$$

Thus :

$$Y_{k,t} = Y_{k,t-1} e^{\mu_k} e^{\varepsilon_{k,t}}$$

where: $Y_{k,t}$ = number of deaths per 1,000 for group k in year t

μ_k = the mean of the transformed series (i.e. logged and differenced series)

$\varepsilon_{k,t}$ = a random error term for group k in year t

Vector Autoregression Model: $m\text{VAR}(p)$

Suppose we have m time series y_{it} , $i = 1, \dots, m$, and $t = 1, \dots, T$ (common length of the time series). Then a vector autoregression model is defined as

$$\begin{pmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{mt} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_m \end{pmatrix} + \begin{pmatrix} \phi_{11}^{(1)} & \phi_{12}^{(1)} & \cdots & \phi_{1m}^{(1)} \\ \phi_{21}^{(1)} & \phi_{22}^{(1)} & \cdots & \phi_{2m}^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{m1}^{(1)} & \phi_{m2}^{(1)} & \cdots & \phi_{mm}^{(1)} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \\ \vdots \\ y_{m,t-1} \end{pmatrix} + \dots + \begin{pmatrix} \phi_{11}^{(p)} & \phi_{12}^{(p)} & \cdots & \phi_{1m}^{(p)} \\ \phi_{21}^{(p)} & \phi_{22}^{(p)} & \cdots & \phi_{2m}^{(p)} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{m1}^{(p)} & \phi_{m2}^{(p)} & \cdots & \phi_{mm}^{(p)} \end{pmatrix} \begin{pmatrix} y_{1,t-p} \\ y_{2,t-p} \\ \vdots \\ y_{m,t-p} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \vdots \\ \epsilon_{mt} \end{pmatrix}.$$