Social Insurance Follow-Up: Methodologies and Implications
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(Slide 2) Presentation Outline

Good morning. My name is Danita Pattemore and I am an Actuary in the Office of the Chief Actuary, Canada. I am pleased to be here today to speak about the use of stochastic processes in the 23rd CPP Actuarial Report and, in particular, its use in the projection of future mortality improvements and life expectancies.

I will start by discussing the stochastic modeling used in CPP 23. Next, I will discuss the methods used in determining future mortality improvements, followed by results in terms of life expectancy and a comparison with life expectancies in the US and the UK. I will conclude with some slides on the sensitivity analysis of mortality improvement rates performed for the report.

(Slide 3) Stochastic Modeling in CPP23

The independent review panel of the 21st CPP Actuarial Report suggested a more extensive use of stochastic processes for future actuarial reports. Ideally, an integrated model would be developed where all parameters are stochastically generated in an integrated fashion. Due to the complexity of the CPP model, this would not be easily achievable and is unrealistic at this time. Instead, in the 23rd Report, stochastic methods were used to project a probability distribution for potential outcomes for key assumptions, such as fertility, investment returns and mortality. These probability distributions were then used to determine appropriate high- and low-cost alternative assumptions for sensitivity testing, as well as the probability that the actual outcome will be within this range. The advantage of stochastic modeling is that future values are described not by unique values, but rather by probability distributions, thus increasing the information available relative to the deterministic model.

(Slide 4) Mortality

The first step in modeling future mortality rates was to subdivide the rates into 40 age-sex groups. The main criteria was to find a model that provided an especially good fit for the age groups 60-64 and 65-69 since the majority of beneficiaries begin receiving their pension at these ages. Once models that met these criteria were identified, the fit was tested for other age groups, especially those above age 60. We initially tried to fit a model to past mortality improvement rates, but this provided very poor fit statistics. Instead, models were fitted to past mortality rates. Future mortality rates were then projected and converted into mortality improvement rates.

(Slide 5) Mortality Improvement Rates: 1927-2004

This graph shows the high variability in male improvement rates for the age group 65-69 during this historical period. This variability made it difficult to fit a time-series model to the data with sufficient fit statistics.

(Slide 6) Mortality Rates: 1926-2004

This graph illustrates the actual number of deaths per 1,000 of males in the age group 65-69 over the historical period. Note that mortality was rather flat until 1974 when significant improvements began to occur.

(Slide 7) Log ARIMA(0,1,0) Model

The time series model selected to reproduce the annual mortality rates is a log ARIMA (0,1,0) which is the difference of consecutive logged terms. This model was selected because the resulting series after logging and differencing consecutive terms is fairly stationary and an analysis of the fit statistics, including R^2, for all age-sex groups indicate that this model provides a very close fit to the actual data. The mean of the data is time-varying, thus it is important to difference the data. Other time series models were tested, but none provided as good a fit as the log ARIMA (0,1,0). In fact, the R^2 value for all age-sex groups was above 0.9. As well, the use of the log transformation eliminates the need for a lower bound of zero since logged mortality rates will always be positive. The addition of autoregressive or moving average terms added complexity to the model without significant improvements to the actual fit. Thus, it was decided to exclude any additional terms.

(Slide 8) Time Series Equation

This is the general time series equation that was used to project future mortality rates. In addition to the historical data, a randomly generated error term is included in the equation so that thousands of future outcomes could be generated.

(Slide 9) Best Estimate

The mortality model is set up to project future mortality rates two different ways. The first method is somewhat deterministic in that the expected value of each scenario is the best-estimate for the entire projection period, as determined by OCA. Stochastic scenarios are then generated such that the confidence interval is centered around this best estimate. At this point in the process, a best estimate had not been determined, so the second methodology was used.

Future mortality projections were allowed to develop without any interference, based on the selected time-series equation and a randomly generated error term based on historical volatility in mortality rates. One thousand scenarios were developed in this way and the expected value is set equal to the median of the generated scenarios.

(Slide 10) OCA Stochastic Model

As mentioned previously, the stochastic model is based on a log ARIMA(0,1,0) time series model. Annual historical mortality rates were calculated for 40 age-sex groups for the period 1926-2004 as the ratio of annual deaths to the population for each group. Data for the annual numbers of deaths and the Canadian population were obtained from Statistics Canada. The first year of data available for analysis is 1926.

Although the mortality rates of one group are not dependent upon the mortality rates of other groups, there is certainly a degree of correlation among groups. This correlation must continue to be reflected in the projected rates and is done so by correlating the error terms of the 40 age-sex groups using Cholesky decomposition.

This model runs 1,000 scenarios and then produces results that include the 95% confidence intervals and medians for mortality rates and mortality improvement rates for each age-sex group. In addition, the median and 95% confidence interval for life expectancy at birth and age 65 were calculated for males and females.

(Slide 11) Stochastic Process

Once the equation was determined and the Cholesky Decomposition performed, future mortality rates were projected for each age-sex group 75 years into the future for 1,000 scenarios. The resulting mortality rate is the median mortality rate over all 1,000 scenarios. In addition, 95% confidence intervals were calculated to create awareness about the range of possible mortality rates.

This chart shows the historical and projected mortality rates for males in the age range 65-69. The middle line represents the median mortality rates of the 1000 scenarios run, while the lines above and below represent the bounds of the 95% confidence interval.

Having projected mortality rates for each age-sex group, the next step was to convert those values into mortality improvement factors. The results tend to show that mortality improvement factors for each age-sex group are rather constant over the projection period with little fluctuation.

(Slide 12) Reasons for Incorporating Judgement

Instead of using the exact mortality improvement factors produced by the model, it was decided to incorporate some judgment in determining the best-estimate mortality improvement factors. The main reason for incorporating judgement is that historical experience is not necessarily reflective of future experience. During the 20th century, structural changes in mortality patterns have lessened the validity of historical experience compared to recent and emerging patterns. An example of this is that historically, female mortality has improved at a faster pace than male mortality; however, males have recently begun to reverse this trend. A fully stochastic model may continue this trend well into the future resulting in male life expectancy reaching and even surpassing female life expectancy. Although OCA believes that the gap between male and female life expectancies will continue to narrow in the near future, we do not believe that male life expectancy will reach that of females. Thus, judgement is required in order to maintain this relationship between male and female life expectancies.

In addition, certain limitations of the ARIMA model make it necessary to incorporate some judgement. The mortality data is logged and then differenced in order to eliminate the time-varying mean. It's possible that this transformation may not completely eliminate the time-varying mean which would lead to understating the degree of uncertainty in the simulated probability distributions of the mortality rates.

Rather than allowing the stochastic model to project future mortality improvements naturally, judgment, along with an analysis of recent trends, is used to set future improvement rates.

(Slide 13) Annual Mortality Improvement Rates

The evolution of a 15-year moving average of historical improvement rates is analyzed through time and then compared to the mortality improvement factors produced by the model in order to finalize the best-estimate mortality improvement factors for each age-sex group. This table summarizes the historical average annual improvement rates for Canada over two 15-year periods. These values are based on central death rates and are similar to results obtained using the Human Mortality Database and the stochastic model.

(Slide 14) Annual Mortality Improvement Rates

Historical improvement rates tend to decline with age and are small or even negative for those aged 90 and over. As we age, it becomes more difficult to improve mortality since death may be the result of multiple medical conditions. Over the last 15 years, both males and females below age 65 have experienced a slowdown in improvement rates. For ages 15 to 64, the annual improvement rate for females has decreased from a level of 2.5% per year in the period 1944-1989 to about 1.5% per year over the last 15 years.

For those aged 65 and older, over the last 30 years a significant slowdown has been observed for females compared to an increase for males. That is, male mortality is improving at a faster rate than female mortality. This explains why the gap in life expectancy between males and females has begun to narrow over the last 30 years.

(Slide 15) Annual Mortality Improvement Rate Assumptions

Best-estimate mortality improvement factors are based on trends over the last 30 years and judgement. During the initial period of 2005-2009, the annual mortality improvement rates are based on actual experience over the last 15 years by age and sex.

The ultimate annual improvement rates for years 2029 and thereafter were derived by trending the experience of female improvement rates over the last 30 years (1974-2004) for another 30 years. Ultimate male improvement rates are assumed to be the same as for females. Since male improvement rates are currently higher than female improvement rates, it is thus implicitly assumed that male mortality will continue to improve at a faster pace than females over the period 2005-2028.

For the intermediate years, a simple linear interpolation is used to determine the annual improvement rates.

(Slide 16) Life Expectancy (without future improvements)

Once the best-estimate mortality improvement factors have been determined, the next step is to apply these factors to the 2001 Canada Life Table (CLT) in order to establish the best-estimate mortality rates for the future. Finally, a stochastic process is used to project 1,000 future mortality rate paths that are centered around this best-estimate. The life expectancy for each of the 1,000 paths is then calculated and the median is considered to be the best-estimate life expectancy. The resulting values are shown in this table under the heading 'Stochastic Process' and are compared to the deterministic results calculated as the best-estimate for the 23rd Actuarial Report. Ideally, these values would be identical. However, limitations of the ARIMA model that were described previously prevent future projections from being perfectly centred around the best-estimate. The result is that the stochastically determined life expectancies at birth are up to half a year less than those computed in the deterministic model, but nevertheless very close at age 65.

(Slide 17) Life Expectancy (with future improvements)

Mortality improvements are expected to continue in the future; therefore it seems reasonable to include all future projected improvements in the life expectancy calculation. Like the previous slide, this table compares life expectancy at birth and age 65 under the two models: stochastic and deterministic, although in this case, improvements are assumed to continue throughout the projection period. As with the previous slide, the stochastically-determined life expectancies at birth are approximately one year less than those in the deterministic model.

(Slide 18) Comparison of Male Life Expectancy at age 65

The next two slides compare life expectancy at age 65 for males and females in Canada, the United States and the United Kingdom. Between 2007 and 2075, male life expectancy in both Canada and the US is projected to increase by approximately 4 years, with Canada maintaining a higher life expectancy in all years. However, in the UK, male mortality is projected to increase by over 4.5 years by 2050, 25 years earlier. The reason that UK life expectancies are so much higher by 2050 is that they assume an annual 1% mortality improvement at all ages.

(Slide 19) Comparison of Female Life Expectancy at age 65

Between 2007 and 2075, female life expectancy in both Canada and the US is projected to increase by approximately 3.5 years. The UK, on the other hand, projects in increase in female life expectancy of 4.5 years by 2050. In addition, all three countries project a narrowing of the gap between male and female life expectancies.

(Slide 20) Sensitivity Test Using Stochastic Process

Although the results determined using the stochastic model were not used in the best-estimate assumptions of the 23rd Actuarial Report, the model itself was very useful for sensitivity testing. The model was used to determine the appropriate range of future life expectancies at birth and age 65 that should be tested. This table shows the life expectancies used in the actuarial report and the upper and lower bound that form the 95% confidence interval. It was projected that, on average, the life expectancy of a male age 65 in 2050 will be in the range 17.8 years to 25.1 years with 95% probability. For a female age 65 in 2050, life expectancy is projected to be in the range 18.6 years to 27.9 years.

(Slide 21) Evolution of the Asset/Expenditure Ratio

An important measure of the CPP's funding status is defined by the ratio of assets at the end of one year to the expenditures of the next year. Under the best-estimate assumptions of the Plan, this ratio is projected to increase over the next two decades, reaching 5.6 by 2025. Thereafter, it rises slowly to a value of 6.0 in 2050 and 6.4 by 2075. This graph demonstrates the impact that mortality rates, other than the best-estimate, could have on the Plan's funding status. The lower and higher life expectancies used in this scenario are stochastically determined and shown in the previous slide

This chart shows the evolution of the asset to expenditure ratio under three scenarios: the best-estimate assumption and the two stochastically determined scenarios based on a 95% confidence interval. The result is that the minimum contribution rate required to finance the plan over a 75-year period could fall between 9.2% and 10.2%.

(Slide 22)

The use of stochastic processes has greatly improved the sensitivity analysis that is performed as part of the triennial actuarial valuations of the Canada Pension Plan. I am pleased to have had to opportunity to speak about the usefulness of stochastic modeling and would be happy to answer any questions you may have. Thank you.