Stochastic Sensitivity Testing for the Canada Pension Plan Actuarial Reports
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Office of the Chief Actuary, Office of the Superintendent of Financial Institutions

Good morning. Thank you for inviting me here today to talk about the stochastic sensitivity testing framework of the Canada Pension Plan actuarial reports.

CPP, a partially funded pension plan *(Slide 2)*

I would like to start with a brief description of the Canada Pension Plan (CPP) – a part of the second tier of the Canadian retirement income system. The CPP is a defined benefit plan providing a retirement benefit equal to 25% of career-average earnings up to a certain maximum. In addition, the Plan provides survivor and disability benefits. The CPP is financed through contributions paid in equal parts by the employer and employees. The contribution rate is currently 9.9% and it is the same since 2003.

The CPP is financed using a partial funding approach called the steady-state funding. This approach complements the pay-as- you-go and full funding approaches of other components of the Canadian retirement income system.

The steady-state funding is a form of optimal financing for the CPP. Its goal is to stabilize the asset/expenditure ratio over time. By doing it the steady-state methodology helps to ensure that the CPP is affordable and sustainable for current and future generations of Canadians. Although the financing methodology could always be changed or reworked altogether, the objective of partially prefunding the Plan should remain paramount.

Under best-estimate assumptions and minimum contribution rate of 9.82%, A/E ratio stabilises at around 5.5 *(Slide 3)*

Along with the stable contribution rate, the Asset/Expenditure ratio is an important measure of the Plan's sustainability – it is the ratio of assets at the end of one year to the expenditures of the next year.

From 2000 to 2019, the net cash flows of the Plan, that is contributions less expenditures, have been and will continue to be positive, resulting in a rapid increase in the Plan's Asset/Expenditure ratio and funding status. These net cash flows are and will continue to be invested by the CPP Investment Board with a view to maximizing the rate of return without undue risk and further increasing the level of pre-funding in the Plan.

This graph demonstrates that with a minimum contribution rate of 9.82%, the Asset/Expenditure ratio, by definition, is stable, at around 5.5 in 2019 and 2069. Since the actual contribution rate of 9.9% is higher than the minimum contribution rate, the blue line shows that the Asset/Expenditure ratio will continue to improve gradually over time.

Uncertainty of results – an important part of the reporting *(Slide 4)*

By law, the statutory Actuarial Reports of the CPP are produced every three years by the Office of the Chief Actuary. The projections prepared for this purpose cover a period of 75 years. They are performed using a deterministic model and "best-estimate" assumptions. The complexity of the model, the number of assumptions involved, as well as the length of the projection period, make it important to communicate to stakeholders limitations of such projections. One of the ways of doing it is through the illustration of the uncertainty of the results presented in the report.

The 23rd CPP actuarial report as of December 31, 2006 contains a range of uncertainty illustrations. This presentation is based on the results on this report.

Scenario and deterministic tests *(Slide 5)*

The first type of sensitivity testing used in the 23rd CPP report is done through scenario tests. Assumptions used for the CPP projections are correlated and their future outcomes are shaped by people behavior as well as by government policies. While some correlation could be estimated using past experience, we feel that we could not attach reliable probability distributions to complex economic and demographic environments. Therefore the scenarios illustrating younger and older population environments were developed using judgement.

On the other hand, stochastic modelling is not practical for illustrating the impact of short-term economic and financial shocks.

The sensitivity tests for individual assumptions are mostly done using stochastic techniques; however for retirement rates, unemployment rates and participation rates they are developed deterministically. The reasons for using deterministic models are in particular, that these assumptions are driven by the Plan' provisions and the society behavioural trends with respect to the labour force participation.

Stochastic sensitivity tests are providing report's users with likelihood of potential outcomes *(Slide 6)*

The goal of using the stochastic modeling for the CPP reports was not to make the whole CPP valuation model stochastic, but to communicate to the CPP stakeholders and other report's readers the likelihood of potential outcomes and associated risks with respect to individual assumptions.

It should be recognized that uncertainty exists in both deterministic and stochastic models. Stochastic models output depends on many factors, some of them being choice of the model, correlation between variables, assumed probability distribution, historical data used to fit the model, length of projection period, etc. We cannot say that the answer is more meaningful because we use a stochastic process. Therefore, it is of utmost importance that the results of such models should be used and communicated with care and clarity. They should be used to educate readers, not to confuse them.

Office of the Chief Actuary (OCA) stochastic tests are based on ARIMA(p,d,q) models *(Slide 7)*

The stochastic models developed by the Office of the Chief Actuary are based on the class of time-series forecasting models called ARIMA(p, d, q) – autoregressive integrated moving average. These models have three parameters with “d” indicating the order of differencing, “p” number of autoregressive terms and “q” number of moving average terms.

The presence of both autoregressive and moving average terms could distort forecast results; thus it is preferable to use either models with only autoregressive terms or moving average terms.

OCA model – combination of stochastic and deterministic elements *(Slide 8)*

Our first step is to fit the appropriate time-series model to the historical data using statistical software. To choose the model we look at the combination of goodness-of-fit statistics. If several models exhibit the similar goodness-of-fit, the model with fewer terms is chosen.

The estimated model’s parameters are then used to run an Excel-based stochastic forecasting tool that runs a large number of scenarios. For each scenario and for each projection year a random error is generated using normal distribution with zero mean and variance-covariance matrix based on the past experience. This error is added to a projected value.

Since our goal is to produce a confidence interval around the best-estimate assumption, we add a deterministic element to our model by forcing the median, or expected value, for each scenario to be equal to the best estimate.

The model generates confidence intervals and cumulative average confidence intervals. The latter are used to determine, or to set, assumptions for high and low cost scenarios.

Model variations for correlated assumptions *(Slide 9)*

If assumptions are considered to be significantly correlated, we use one of the two following methodologies. Under the first one, a model for each assumption is fit separately, but the correlation between assumptions is reflected in projected rates by correlating the error terms. An example of this technique is our model of mortality rates grouped by gender and age and it will be discussed later in the presentation.

Under the second methodology we use vector autoregressive modeling to fit several assumptions simultaneously. In stochastic forecast each projected variable depends on previous values of all variables and, once again, error terms are correlated. We model rate of return on asset classes and inflation using this approach.

More details on Cholesky decomposition used to correlate error terms and vector autoregressive modeling could be found in the Appendix.

The remaining part of this presentation will illustrate the results of stochastic sensitivity testing for three assumptions, the ones that produce the highest volatility in the contribution rate: fertility, mortality and rate of return on assets.

Fertility – 95% confidence interval *(Slide 10)*

This slide shows the historical Canadian fertility rate (the dotted line), the best-estimate fertility rate assumption as well as lower and upper bound of the 95% cumulative average confidence interval obtained using autoregressive model with four terms.

By 2079, the lower and upper bounds of this confidence interval are respectively, 1.1 and 2.1 births per women, and the interval is symmetric around the best-estimate assumption of 1.6 births. The chart also illustrates the dependence of model outputs on the length of the projection horizon: if, for example, a 10-year projection period is considered, then the range becomes wider at 0.9 to 2.3.

Under lower fertility scenario assets are depleted by 2069 *(Slide 11)*

As it was mentioned earlier, an important measure of the CPP's sustainability is defined by the ratio of assets at the end of one year to the expenditures of the next year. This chart shows the evolution of asset/expenditure ratio with the legislated contribution rate of 9.9% under the best-estimate assumptions and for the low and high fertility assumptions shown on the previous slide.

It could be seen that under the high-cost scenario, that is low fertility rate, the CPP fund would be depleted by 2069. So in order to stabilize the asset/expenditure ratio, the minimum contribution rate should be increased to 10.45%. Under high-fertility or low-cost scenario, the minimum contribution rate would become 9.25%.

By including this information in the report, the OCA provides CPP stakeholders with a feeling of what the contribution rates might be if the average fertility rate ends up being within 95% confidence interval.

Mortality model output: confidence intervals for life expectancies *(Slide 12)*
One of the most important assumptions for the CPP is the mortality rates assumption and, in particular, future mortality improvement rates.

The first step in modeling sensitivity for future mortality rates was to subdivide the rates into 40 age-sex groups. We initially tried to fit a model to past mortality improvement rates, but this provided very poor fit statistics. Instead, we first transform the past mortality rates using logarithmic function. This is done in order to avoid negative mortality rates. Then we fit simple ARIMA models to these transformed rates. In other words we use logARIMA models. The correlations between age-sex groups were modelled using Cholesky decomposition for error terms. Future mortality rates, centred around the best-estimate, are then projected and converted into mortality improvement rates. More details on the model could be found in the Appendix.

Sensitivity test using stochastic process *(Slide 13)*
This table shows the life expectancies with future mortality improvements produced by the stochastic methodology; that is the median or expected life expectancy, and the upper and lower bound that form the 95% confidence interval. It was projected that, on average, the life expectancy of a male age 65 in 2050 will be in the range of 17.8 years to 25.1 years with 95% probability. For a female age 65 in 2050, life expectancy is projected to be in the range of 18.6 years to 27.9 years.

It is interesting to note that the 95% confidence interval is not symmetric with respect to the median. The reason is that people could not be expected to live indefinitely; therefore positive mortality rates as well as the limit age of 120 years are used.

Although mortality rates could not decrease forever, their future trajectory could require an increase in the contribution rate *(Slide 14)*

This graph demonstrates the impact that mortality rates, other than the best-estimate, could have on the Plan’s asset/expenditure ratio. The lower and higher life expectancies used in this scenario are the ones shown on the previous slide.

As discussed, it could be seen that the picture is not symmetric, with the high-cost scenario being closer to the best estimate. Due to the uncertainty in future mortality, the minimum contribution rate, required to finance the plan over a 75-year period, is projected to be in the range between 9.2% and 10.2%.

Real rate of return of portfolio – four correlated variables *(Slide 15)*
Given the partial funding nature of the CPP, it is important to illustrate the CPP exposure to the investment risk.

We use vector autoregressive model with two terms to fit simultaneously four assumptions: rates of return on Canadian equities, foreign equities and fixed income, and inflation. The projected rates of return and inflation are used to determine the 95% confidence interval for the real rate of return on the CPP assets.

The obtained 95% confidence interval has a width of 3% and is centered around the best-estimate assumption of the real rate of return of 4.2%.

Real rate of return sensitivity testing exhibits the highest volatility in the contribution rate…even for a partially funded plan *(Slide 16)*
This slide shows that the real rate of return assumption produces the highest volatility in the asset/expenditure ratio and in the minimum contribution rate: it is projected that this rate could be between 9.0% and 10.7%. Even for a partially funded plan, where the investment risk is mitigated by the existence of the pay-as-you-go component, the impact is quite dramatic. The volatility in contribution rate faced by a fully funded plan will be much higher.

This example illustrates the fact that if the objective of a defined benefit social security system is the stability of the contribution rate, the full funding financing method is not the optimal financing path.

Conclusions and next steps *(Slide 17)*
The application of stochastic processes in the social security modeling is a long journey, not a destination. And we are at the beginning of this journey. As technology develops, new possibilities arise. As plans mature, more historical data is available.

In accordance with the recommendation of the independent peer review panel, the OCA will continue to develop its stochastic models. Since we think that in Canada there is a valid correlation between fertility and migration, the next step on our wish list is to try to model simultaneously these two assumptions.

Finally, let me repeat myself and say again that the stochastic modeling is a tool, not a crystal ball, and as any tool it should be used wisely.

Thank you for your attention.